above a suddenly heated wire can be seen in the photographs on p. 347 of [5]. These observations strengthen the assumption that $\bar{R} \sim 1100$ at t^*. The sensitivity of the interferometric method with the fluids used in the present study was insufficient to permit a detailed investigation of the structure of these cells.

CONCLUSIONS

Experimental measurements of the delay time during which heat transfer from a suddenly heated fine horizontal wire to a fluid is essentially by conduction agree rather well with those predicted by equation (4). This correlation suggests that the onset of significant convection above a heated horizontal wire is associated with a fluid instability in the sense that there is a critical value of an appropriately defined Rayleigh number below which disturbances grow slowly with respect to the characteristic thermal diffusion time of the fluid. Above this critical value disturbances grow rapidly and initiate significant convection. The expression (4) for the delay time may also be useful as a guide in the design of experiments to determine thermal diffusivities by transient response measurements, and for the interpretation of experimental data regarding onset of boiling and other heat-transfer phenomena near suddenly heated wires.

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TO GEOMETRIZED THEORY OF HYPERBOLIC HEAT CONDUCTION EQUATION

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1. IT IS well known that a linear parabolic heat-conduction equation leads to the paradox of infinite velocity of propagation of heat disturbances. All attempts to solve this paradox have revealed the necessity to consider a hyperbolic heat-transfer equation. A number of papers [1–5] are concerned with the derivation and substantiation of this equation. A. V. Luikov [1, 2] has found that an equation of the form

$$c_\gamma \frac{\partial T}{\partial t} + \frac{\lambda}{w^2}\frac{\partial^2 T}{\partial t^2} = \lambda \nabla^2 T + p \qquad (1)$$

holds for capillary-porous systems, if a finite velocity of heat and mass propagation is assumed. In equation (1) w is the heat propagation velocity $w = \sqrt{(\lambda/c_\gamma \tau)}$, τ is the relaxation time, p is the heat source function, λ is the thermal conductivity, c_γ is the volumetric heat capacity.

In case of small c_γ and large mean free molecular path lengths the first term in the right hand side of equation (1) is small compared to the other terms and may therefore be omitted. The wave form of the heat-conduction equation is obtained as a result

$$\frac{\partial^2 T}{\partial t^2} = w^2 \nabla^2 T + \frac{w^2}{\lambda} p. \qquad (2)$$

At present some experimental works are available which confirm a wave nature of heat transfer [7].

A geometrical approach involving Riemannian manifolds to composition of differential equations, particularly those describing heat-transfer processes was developed by A.S. Predvoditelev [6].

In the present communication a geometrical interpretation of heat-transfer processes governed by equation (2) is developed. This trend follows from the works by Poincaré [8], Barankin [9], Robertson [10].

2. Nonuniformly heated medium with the temperature $T(x, y, z, t)$ induces a Reimannian space with a metric of the form

$$ds^2 = \frac{1}{(1 + hT)^2}\,[w^2dt^2 - (dx^2 + dy^2 + dz^2)]. \tag{3}$$

The metric described by expression (3) is equivalent to the invariant $ds^2 = w^2dt^2 - dx^2 - dy^2 - dz^2$ of the acoustic theory of relativity [11]. In the metric the sound velocity w is taken as a fundamental velocity. Expression (3) implies that the lengths are measured by a calibration rod with thermal expansion coefficient h.

The Riemannian space with a metric of the form (3) may be described by mean curvature $\bar{K}$. As is known from the

$$\bar{K} = \frac{h}{2w^2}\Box^2 T + O(h^2) \tag{4}$$

where $\Box^2$ is the d'Alembert operator in equation (2).

With the help of equation (2) a final form of the expression for a mean curvature is obtained

$$\bar{K} = \frac{hp}{2\lambda}. \tag{5}$$

Thus, the heat transfer problem for hyperbolic equation (2) is reduced to finding a mean curvature of the Riemannian space with metric (3). As follows from equation (5), the curvature of this space is proportional to the function of heat sources. This is similar to curving of the space caused by the material mass effect in the Einstein gravitation theory.

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REYNOLDS FLUX AND DANCKWERTS SURFACE RENEWAL THEORY

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NOMENCLATURE

A, dA,	area of surface;
C,	concentration;
C_p,	specific heat at constant pressure;
D,	diffusion coefficient;
h,	heat-transfer coefficient;